

Exercise 73

If $F(x) = f(3f(4f(x)))$, where $f(0) = 0$ and $f'(0) = 2$, find $F'(0)$.

Solution

Use the chain rule to differentiate $F(x)$.

$$\begin{aligned} F'(x) &= \frac{d}{dx}[F(x)] \\ &= \frac{d}{dx}[f(3f(4f(x)))] \\ &= f'(3f(4f(x))) \cdot \frac{d}{dx}[3f(4f(x))] \\ &= f'(3f(4f(x))) \cdot 3f'(4f(x)) \cdot \frac{d}{dx}[4f(x)] \\ &= f'(3f(4f(x))) \cdot 3f'(4f(x)) \cdot 4f'(x) \end{aligned}$$

Evaluate it at $x = 0$.

$$\begin{aligned} F'(0) &= f'(3f(4f(0))) \cdot 3f'(4f(0)) \cdot 4f'(0) \\ &= f'(3f(4(0))) \cdot 3f'(4(0)) \cdot 4(2) \\ &= f'(3f(0)) \cdot 3f'(0) \cdot (8) \\ &= f'(3(0)) \cdot 3(2) \cdot (8) \\ &= f'(0) \cdot (6) \cdot (8) \\ &= (2) \cdot (6) \cdot (8) \\ &= 96 \end{aligned}$$